Problem 12-6

The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint*: Plot the path to determine the total distance traveled.

Solution

The position of the particle at $t = 6$ s is

$$s(6) = 1.5(6)^3 - 13.5(6)^2 + 22.5(6) = -27 \text{ ft}.$$

Differentiate the given position function to get the velocity.

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{d}{dt}(1.5t^3 - 13.5t^2 + 22.5t) \\ &= 1.5(3)t^2 - 13.5(2)t + 22.5 \\ &= 4.5t^2 - 27t + 22.5 \end{aligned}$$

The total distance the particle travels from $t = 0$ s to $t = 6$ s is given by the integral of the speed over this interval. Note that $v(t)$ is negative from $t = 1$ to $t = 5$ and positive elsewhere.

$$\begin{aligned} s_T &= \int_0^6 |v(t)|\, dt \\ &= \int_0^6 |4.5t^2 - 27t + 22.5|\, dt \\ &= \int_0^1 (4.5t^2 - 27t + 22.5)\, dt + \int_1^5 (-4.5t^2 + 27t - 22.5)\, dt + \int_5^6 (4.5t^2 - 27t + 22.5)\, dt \\ &= \left(\frac{4.5}{3}t^3 - \frac{27t^2}{2} + 22.5t\right)\Bigg|_0^1 + \left(-\frac{4.5}{3}t^3 + \frac{27t^2}{2} - 22.5t\right)\Bigg|_1^5 + \left(\frac{4.5}{3}t^3 - \frac{27t^2}{2} + 22.5t\right)\Bigg|_5^6 \\ &= (10.5) + (48) + (10.5) \\ &= 69 \text{ ft} \end{aligned}$$